Coordinate Geometry for AMC 10/12

Essential Analytical Methods for AIME Qualification

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Contents

1 Introduction

On the AMC 10 and AMC 12, coordinate geometry is one of the most reliable tools. While synthetic Euclidean geometry requires finding specific auxiliary lines or hidden angle-chasing patterns, setting up a coordinate system converts geometric relationships directly into algebra. If a problem involves right angles, squares, rectangles, midpoint divisions, or simple ratios, coordinate geometry can often solve it directly without needing any clever geometric tricks.

However, using coordinates effectively on a timed test like the AMC requires judgment. If you set up an inefficient coordinate frame, you risk running into long, messy calculations that consume valuable time. The goal is to set up your axes strategically, so that equations remain simple, and calculations stay manageable under time constraints.

When deciding whether to use coordinates on an AMC problem, ask yourself:

- Does the diagram contain clear perpendicular lines or right angles that naturally form an x-y axis?
- Are the key geometric figures squares, rectangles, right triangles, or circles centered at predictable locations?
- Can simple integer or single-variable coordinates be assigned to all critical points in the problem?
- Will the solution path lead to nasty radicals or a neat algebraic solution?

2 Strategic Coordinate Placement

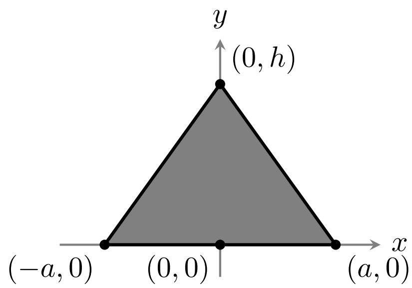
Choosing where to place the origin $(0, 0)$ and how to align the x- and y-axes are crucial for a succesful coordinate solution.

Avoid automatically placing the origin at the bottom-left corner of a figure. Instead, look for points of high symmetry, midpoints of key segments, or vertices where multiple perpendicular lines intersect.

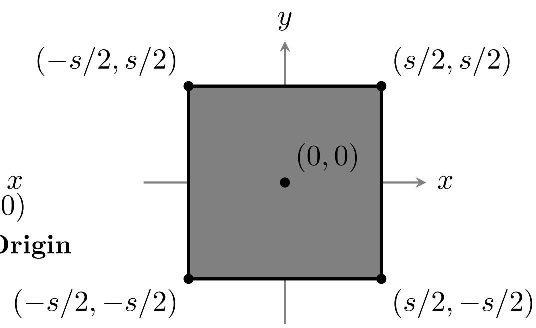
Axis Placement Rules for the AMC

Use these placement strategies based on the figure given:

- **Right Triangles:** Place the origin at the right-angled vertex $C(0, 0)$. The legs lie along the positive axes, giving vertices $A(0, b)$ and $B(a, 0)$.
- **Isosceles Triangles:** Place the origin at the midpoint of the base. If the base has length $2a$ and height h, the vertices become $(-a, 0)$, $(a, 0)$, and $(0, h)$.
- **Squares and Rectangles:** Place the origin at the center $(0, 0)$ if the problem is highly symmetric, or at the bottom-left vertex if you want to avoid negative numbers.
- **Circles:** Place the origin at the center of the circle so its equation becomes $x^2 + y^2 = R^2$.



Isosceles Triangle: Midpoint Origin



Symmetric Square: Center Origin

Example 2.1

Let $ABCD$ be a square of side length 6. Point M is the midpoint of AB, and point N lies on BC such that $BN = 2$. Find the area of triangle DMN.

Solution. Place the square in the first quadrant with $A(0,6)$, $B(6,6)$, $C(6,0)$, and $D(0,0)$.

Using the given ratios, find the coordinates of each point:

- Point D is at $(0,0)$.
- Point M, the midpoint of AB, is at $(3,6)$.
- Point N lies on BC with $BN = 2$. Moving 2 units down from $B(6,6)$ gives $N(6,4)$.

To compute the area of $\triangle DMN$, subtract the areas of the three surrounding right triangles from the area of the square (36):

- Area of $\triangle ADM = \frac{1}{2} \times 6 \times 3 = 9$.
- Area of $\triangle MBN = \frac{1}{2} \times 3 \times 2 = 3$.
- Area of $\triangle DCN = \frac{1}{2} \times 6 \times 4 = 12$.

Subtracting these areas from the total area of the square gives:

$$\text{Area}(\triangle DMN) = 36 - (9 + 3 + 12) = 36 - 24 = 12.$$

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3 Essential AMC Formulas

Speed on the AMC requires memorizing foundational line and distance formulas so you can apply them instantly.

3.1 Distance and Midpoint

The distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The midpoint M of segment P_1P_2 is:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

3.2 Section Formula (Internal Division)

If point P divides the segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$, then:

$$P = \left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n} \right).$$

3.3 Point-to-Line Distance

The shortest (perpendicular) distance from a point $P(x_0, y_0)$ to a line $Ax + By + C = 0$ is:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}.$$

3.4 Shoelace Formula for Polygon Area

For a polygon with ordered vertices $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, the area is:

$$\text{Area} = \frac{1}{2}|(x_1y_2 + x_2y_3 + \cdots + x_ny_1) - (y_1x_2 + y_2x_3 + \cdots + y_nx_1)|.$$

Example 3.1

Find the distance from the point $(3, 5)$ to the line $4x - 3y + 8 = 0$.

Solution. Apply the point-to-line distance formula with $x_0 = 3$, $y_0 = 5$, $A = 4$, $B = -3$, and $C = 8$:

$$d = \frac{|4(3) - 3(5) + 8|}{\sqrt{4^2 + (-3)^2}} = \frac{|12 - 15 + 8|}{\sqrt{25}} = \frac{5}{5} = 1.$$

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4 Circles and Power of a Point

Circles frequently appear on both the AMC 10 and AMC 12. The standard equation of a circle centered at (h, k) with radius R is:

$$(x - h)^2 + (y - k)^2 = R^2.$$

4.1 Power of a Point in Coordinates

For a circle ω given by $(x - h)^2 + (y - k)^2 = R^2$, the power of a point $P(x_0, y_0)$ relative to ω is:

$$\text{Pow}_\omega(P) = (x_0 - h)^2 + (y_0 - k)^2 - R^2.$$

If P is outside the circle, this power equals the square of the length of the tangent segment drawn from P to ω.

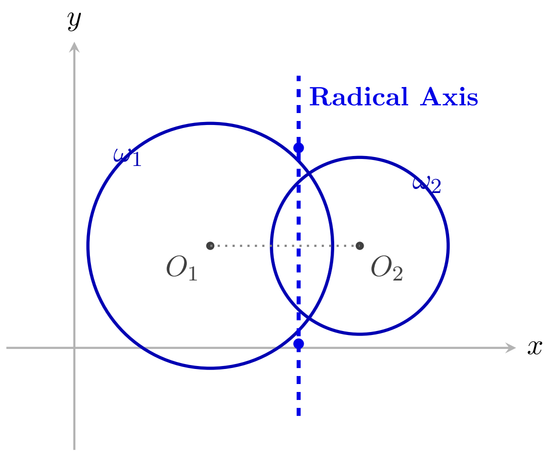
4.2 The Radical Axis of Two Circles

The radical axis of two non-concentric circles is the line consisting of all points that have equal power with respect to both circles. For two circles:

$$\omega_1 : x^2 + y^2 + D_1x + E_1y + F_1 = 0, \qquad \omega_2 : x^2 + y^2 + D_2x + E_2y + F_2 = 0,$$

subtracting their equations cancels out the quadratic $x^2 + y^2$ terms, yielding the linear equation of the radical axis:

$$(D_1 - D_2)x + (E_1 - E_2)y + (F_1 - F_2) = 0.$$



If the two circles intersect at two points, their radical axis is simply the line passing through those two intersection points.

Example 4.1

Find the equation of the radical axis of the circles $x^2 + y^2 - 4x - 2y + 1 = 0$ and $x^2 + y^2 + 2x - 6y + 6 = 0$.

Solution. Subtract the second equation from the first equation:

$$(x^2 + y^2 - 4x - 2y + 1) - (x^2 + y^2 + 2x - 6y + 6) = 0$$

$$-6x + 4y - 5 = 0 \implies 6x - 4y + 5 = 0.$$

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5 Conic Sections for the AMC 12

On the AMC 12, conic sections are not a focused topic, but they can be useful to solve many problems. Last year, on the AMC 12A, problem 14 directly addressed an ellipse: a surprise to many including me. It seems like conics are becoming more common the coming years, so mastering the basic structures can be useful.

5.1 Parabolas

A parabola is the set of points equidistant from a focus $(0, p)$ and a directrix line $y = -p$. Its standard vertical equation is:

$$x^2 = 4py \quad \text{or} \quad y = ax^2.$$

The distance from the vertex to the focus is $|p| = \frac{1}{|4a|}$. The chord passing through the focus perpendicular to the axis of symmetry is the **latus rectum**, with total length $|4p|$. Most AMC problems will question understanding on these fancy terms. Rather, problems frequently assess mastery with one core concept: equal distance between the focus and directrix for all points.

5.2 Ellipses

An ellipse centered at the origin with horizontal major axis length $2a$ and vertical minor axis length $2b$ has the equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

The foci are located at $(\pm c, 0)$, where $c^2 = a^2 - b^2$. For any point P on the ellipse, the sum of the distances to the foci is constant: $PF_1 + PF_2 = 2a$. The **eccentricity** is $e = \frac{c}{a} < 1$.

5.3 Hyperbolas

A hyperbola centered at the origin opening horizontally has the equation:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

The foci lie at $(\pm c, 0)$, where $c^2 = a^2 + b^2$. For any point P on the hyperbola, $|PF_1 - PF_2| = 2a$. The equations of its asymptotes are $y = \pm \frac{b}{a}x$, and the eccentricity is $e = \frac{c}{a} > 1$.

Tangent Lines to Conics

To quickly find the line tangent to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at a specific point (x_0, y_0) on the curve, split the squared terms:

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1.$$

Similarly, for a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the tangent line at (x_0, y_0) is $\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1$.

Example 5.1

Find the foci and asymptotes of the hyperbola $9x^2 - 16y^2 = 144$.

Solution. Divide by 144 to put the equation in standard form:

$$\frac{x^2}{16} - \frac{y^2}{9} = 1.$$

Here $a^2 = 16 \implies a = 4$ and $b^2 = 9 \implies b = 3$.

- Find c: $c^2 = a^2 + b^2 = 16 + 9 = 25 \implies c = 5$. The foci are at $(\pm 5, 0)$.
- Asymptotes: $y = \pm \frac{b}{a}x \implies y = \pm \frac{3}{4}x$.

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6 Problem Set

6.1 Introductory AMC 10 Level Problems

1. Find the equation of the line that passes through $(1, 4)$ and is perpendicular to $2x - 3y = 6$.
2. Compute the area of the triangle with vertices at $(0, 0)$, $(5, 2)$, and $(2, 6)$.
3. Find the center and radius of the circle $x^2 + y^2 - 8x + 10y + 5 = 0$.
4. Point A is at $(1, 2)$ and point B is at $(7, 10)$. Find the coordinates of point P on segment AB such that $AP : PB = 1 : 2$.
5. **(AMC 10)** A square has vertices at $(0, 0)$, $(4, 0)$, $(4, 4)$, and $(0, 4)$. A line passing through $(1, 0)$ divides the area of the square into two equal halves. Find the slope of this line.
6. Compute the distance between the parallel lines $3x + 4y = 10$ and $3x + 4y = 25$.
7. Find the point on the line $y = 2x + 1$ that is closest to the origin $(0, 0)$.
8. **(AMC 10)** A circle centered at $(2, 3)$ is tangent to the line $3x + 4y = 3$. Find the area of the circle.
9. Find the y-intercept of the line that passes through the midpoint of $(2, 8)$ and $(6, 2)$ and is parallel to $x + 2y = 5$.
10. Find the area of the quadrilateral with vertices $(1, 1)$, $(5, 2)$, $(4, 6)$, and $(0, 4)$.

6.2 Intermediate AMC 10/12 Level Problems

11. **(AMC 12 2015)** A circle passes through the points $(0, 0)$, $(0, 6)$, and $(8, 0)$. Find the equation of the line tangent to this circle at the origin.
12. **(AMC 10 2018)** A line with positive slope passes through the origin and is tangent to the circle $(x - 5)^2 + y^2 = 9$. Find the equation of this line.
13. **(AMC 12 2012)** Find the distance between the two points of intersection of the line $y = 2x + 1$ and the circle $x^2 + y^2 - 4x - 2y - 5 = 0$.
14. **(AMC 10 2020)** Let R be the rectangle with vertices $(0, 0)$, $(8, 0)$, $(8, 6)$, and $(0, 6)$. A line with slope m divides R into two polygons of equal area. If the line passes through $(2, 1)$, find m.
15. Find the radical axis of the two circles $x^2 + y^2 - 6x - 4y + 9 = 0$ and $x^2 + y^2 - 2x + 4y + 1 = 0$.
16. **(AMC 12 2014)** A parabola has vertex $(2, 3)$ and directrix $y = 1$. Find the coordinates of its focus and the equation of the parabola.
17. **(AMC 12 2017)** An ellipse is centered at $(0, 0)$ with a horizontal major axis of length 10. If the distance between its foci is 8, find the equation of the ellipse.
18. Find the area of the region bounded by the graph $|x| + |y| = 4$.
19. **(AMC 10 2016)** A circle of radius 2 has its center on the line $y = x$. If the circle is tangent to the x-axis, find all possible x-coordinates of its center.

20. Point $P(x, y)$ moves such that its distance to $(3, 0)$ is twice its distance to the y-axis. Show that the locus of P is a hyperbola and find its standard equation.

6.3 Targeted AMC 12 Conic Problems

21. **(AMC 12 2019)** A hyperbola has asymptotes $y = 2x$ and $y = -2x$, and passes through the point $(3, 4)$. Find the equation of the hyperbola and the distance between its foci.

22. Find the equations of all lines passing through $(4, -2)$ that are tangent to the circle $x^2 + y^2 = 10$.

23. **(AMC 12 2021)** An ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through $(2, 3)$ and has a focus at $(\sqrt{5}, 0)$. Find $a^2 + b^2$.

24. Find the locus of points (x, y) such that the distance from (x, y) to the point $(0, 2)$ is equal to its distance to the line $y = -2$.

25. Find the area of the triangle formed by the line $x + 2y = 6$ and the coordinate axes.

26. Prove that for any point P on the rectangular hyperbola $xy = c^2$, the area of the triangle formed by the tangent line at P and the coordinate axes is constant.

27. **(AMC 12)** A line L with slope $m > 0$ passes through the focus of the parabola $y^2 = 8x$ and intersects the parabola at points A and B. If the length of segment AB is 18, find m.

28. Find the standard equation of the parabola with focus $(0, 4)$ and directrix $y = -4$.

7 Solutions and Answers

7.1 Introductory Solutions

1. Slope of $2x - 3y = 6$ is $\frac{2}{3}$. The perpendicular slope is $-\frac{3}{2}$. Using point-slope form: $y - 4 = -\frac{3}{2}(x - 1) \implies 3x + 2y = 11$.

2. Apply the Shoelace Formula with vertices $(0, 0)$, $(5, 2)$, and $(2, 6)$:

$$\text{Area} = \frac{1}{2}|(0 \cdot 2 + 5 \cdot 6 + 2 \cdot 0) - (0 \cdot 5 + 2 \cdot 2 + 6 \cdot 0)| = \frac{1}{2}|30 - 4| = 13.$$

3. Complete the square: $(x - 4)^2 - 16 + (y + 5)^2 - 25 + 5 = 0 \implies (x - 4)^2 + (y + 5)^2 = 36$. Center is $(4, -5)$, radius is 6.

4. Use the section formula with ratio $1 : 2$:

$$P = \left(\frac{2(1) + 1(7)}{1 + 2}, \frac{2(2) + 1(10)}{1 + 2}\right) = \left(\frac{9}{3}, \frac{14}{3}\right) = \left(3, \frac{14}{3}\right).$$

5. Any line dividing a square's area in half must pass through its center $(2, 2)$. The line passes through $(1, 0)$ and $(2, 2)$, so its slope is $m = \frac{2-0}{2-1} = 2$.

6. Pick a point on $3x + 4y = 10$, such as $(2, 1)$. Compute its distance to $3x + 4y - 25 = 0$:

$$d = \frac{|3(2) + 4(1) - 25|}{\sqrt{3^2 + 4^2}} = \frac{|-15|}{5} = 3.$$

7. The line perpendicular to $y = 2x + 1$ passing through the origin is $y = -\frac{1}{2}x$. Intersect them:

$$2x + 1 = -\frac{1}{2}x \implies \frac{5}{2}x = -1 \implies x = -\frac{2}{5}, \quad y = \frac{1}{5}.$$

The closest point is $\left(-\frac{2}{5}, \frac{1}{5}\right)$.

8. Radius R is the distance from $(2, 3)$ to $3x + 4y - 3 = 0$:

$$R = \frac{|3(2) + 4(3) - 3|}{\sqrt{3^2 + 4^2}} = \frac{|15|}{5} = 3.$$

The area of the circle is $\pi R^2 = 9\pi$.

9. Midpoint of $(2, 8)$ and $(6, 2)$ is $\left(\frac{2+6}{2}, \frac{8+2}{2}\right) = (4, 5)$. The slope of $x + 2y = 5$ is $-\frac{1}{2}$. The parallel line through $(4, 5)$ is $y - 5 = -\frac{1}{2}(x - 4) \implies y = -\frac{1}{2}x + 7$. The y-intercept is 7.

10. Using the Shoelace Formula with $(1, 1), (5, 2), (4, 6), (0, 4)$:

$$\text{Area} = \frac{1}{2}|(1{\cdot}2+5{\cdot}6+4{\cdot}4+0{\cdot}1)-(1{\cdot}5+2{\cdot}4+6{\cdot}0+4{\cdot}1)| = \frac{1}{2}|(2+30+16)-(5+8+4)| = \frac{1}{2}|48-17| = \frac{31}{2}.$$

7.2 Intermediate Solutions

11. The points form a right triangle with vertices $(0, 6)$, $(0, 0)$, and $(8, 0)$. The hypotenuse connects $(0, 6)$ and $(8, 0)$, so the circle's center is its midpoint $(4, 3)$. The radius vector from $(4, 3)$ to $(0, 0)$ has slope $\frac{3}{4}$. The tangent line at $(0, 0)$ is perpendicular to this radius vector, so its slope is $-\frac{4}{3}$. Its equation is $y = -\frac{4}{3}x \implies 4x + 3y = 0$.

12. Let the line be $y = mx$. The distance from the center $(5, 0)$ to $mx - y = 0$ equals the radius 3:

$$\frac{|5m|}{\sqrt{m^2 + 1}} = 3 \implies 25m^2 = 9(m^2 + 1) \implies 16m^2 = 9 \implies m = \frac{3}{4} \quad (m > 0).$$

The equation of the line is $y = \frac{3}{4}x \implies 3x - 4y = 0$.

13. Complete the square for the circle: $(x - 2)^2 + (y - 1)^2 = 10$. Substitute $y = 2x + 1$:

$$(x - 2)^2 + (2x)^2 = 10 \implies x^2 - 4x + 4 + 4x^2 = 10 \implies 5x^2 - 4x - 6 = 0.$$

Let the roots be x_1, x_2. Then $x_1 + x_2 = \frac{4}{5}$ and $x_1 x_2 = -\frac{6}{5}$, so $(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1 x_2 = \frac{16}{25} + \frac{24}{5} = \frac{136}{25}$. Since $y_1 - y_2 = 2(x_1 - x_2)$, the total distance is:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{5(x_1 - x_2)^2} = \sqrt{5 \cdot \frac{136}{25}} = \frac{\sqrt{680}}{5} = \frac{2\sqrt{170}}{5}.$$

14. Any line dividing a rectangle's area in half must pass through its center $(4, 3)$. The line passes through $(2, 1)$ and $(4, 3)$, so its slope is $m = \frac{3-1}{4-2} = 1$.

15. Subtract the two circle equations:

$$(x^2 + y^2 - 6x - 4y + 9) - (x^2 + y^2 - 2x + 4y + 1) = 0 \implies -4x - 8y + 8 = 0 \implies x + 2y = 2.$$

16. The focal length p is the distance from the vertex $(2,3)$ to the directrix $y=1$, so $p=2$. The focus is p units above the vertex at $(2,5)$. The equation of the parabola opening upward is $(x-2)^2 = 4(2)(y-3) \implies (x-2)^2 = 8(y-3)$.

17. Major axis $2a = 10 \implies a = 5$. Distance between foci $2c = 8 \implies c = 4$. Using $b^2 = a^2 - c^2 = 25 - 16 = 9$, the equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

18. The graph $|x| + |y| = 4$ forms a square rotated by $45°$ with vertices at $(4,0)$, $(0,4)$, $(-4,0)$, and $(0,-4)$. The diagonal length is 8, so its area is $\frac{1}{2} \times 8 \times 8 = 32$.

19. Since the center lies on $y = x$, let it be (c,c). Tangency to the x-axis means the radius equals $|c|$. We are given the radius is 2, so $|c| = 2 \implies c = 2$ or $c = -2$. The possible x-coordinates are $x = 2$ and $x = -2$.

20. Setting distance to $(3,0)$ equal to twice the distance to $x = 0$:

$$\sqrt{(x-3)^2 + y^2} = 2|x| \implies (x-3)^2 + y^2 = 4x^2 \implies 3x^2 + 6x - y^2 = 9.$$

Complete the square: $3(x+1)^2 - y^2 = 12 \implies \frac{(x+1)^2}{4} - \frac{y^2}{12} = 1$. This is a hyperbola.

7.3 Targeted AMC 12 Solutions

21. Asymptotes $y = \pm 2x$ mean the equation has the form $4x^2 - y^2 = C$. Plug in $(3,4)$:

$$4(3)^2 - 4^2 = 36 - 16 = 20 \implies 4x^2 - y^2 = 20 \implies \frac{x^2}{5} - \frac{y^2}{20} = 1.$$

Here $a^2 = 5$ and $b^2 = 20$, so $c^2 = a^2 + b^2 = 25 \implies c = 5$. The distance between foci is $2c = 10$.

22. Let the line be $y + 2 = m(x-4) \implies mx - y - (4m+2) = 0$. Setting its distance to $(0,0)$ equal to $\sqrt{10}$:

$$\frac{|-(4m+2)|}{\sqrt{m^2+1}} = \sqrt{10} \implies (4m+2)^2 = 10(m^2+1) \implies 6m^2+16m-6 = 0 \implies 3m^2+8m-3 = 0.$$

Factoring gives $(3m-1)(m+3) = 0$, so $m = \frac{1}{3}$ or $m = -3$. The tangent lines are $x - 3y = 10$ and $3x + y = 10$.

23. Focus at $(\sqrt{5}, 0) \implies c^2 = a^2 - b^2 = 5 \implies a^2 = b^2 + 5$. Substitute $(2,3)$ into the ellipse equation:

$$\frac{4}{b^2+5} + \frac{9}{b^2} = 1 \implies 4b^2 + 9(b^2+5) = b^2(b^2+5) \implies b^4 - 8b^2 - 45 = 0.$$

Factoring gives $(b^2 - 15)(b^2 + 3) = 0 \implies b^2 = 15$. Then $a^2 = 15 + 5 = 20$, so $a^2 + b^2 = 20 + 15 = 35$.

24. Distance from (x,y) to $(0,2)$ equals distance to $y = -2$:

$$\sqrt{x^2 + (y-2)^2} = |y+2| \implies x^2 + y^2 - 4y + 4 = y^2 + 4y + 4 \implies x^2 = 8y.$$

This locus is the parabola $y = \frac{1}{8}x^2$.

25. The line $x + 2y = 6$ has x-intercept $(6, 0)$ and y-intercept $(0, 3)$. The triangle is right-angled at the origin, so its area is $\frac{1}{2} \times 6 \times 3 = 9$.

26. Let $P = \left(t, \frac{c^2}{t}\right)$. The slope of the tangent line at P is $y' = -\frac{c^2}{t^2}$. The equation of the tangent line is:
$$y - \frac{c^2}{t} = -\frac{c^2}{t^2}(x - t) \implies \frac{c^2}{t^2}x + y = \frac{2c^2}{t}.$$
The x-intercept is $A(2t, 0)$ and the y-intercept is $B\left(0, \frac{2c^2}{t}\right)$. The area of $\triangle OAB$ is:
$$\text{Area} = \frac{1}{2}(2t)\left(\frac{2c^2}{t}\right) = 2c^2,$$
which is constant and independent of t.

27. For $y^2 = 8x$, we have $4p = 8 \implies p = 2$, so the focus is $F(2, 0)$. A line through F with slope m is $y = m(x - 2)$. By the focal chord property, the focal chord length for $y^2 = 4px$ is $AB = 4p\csc^2\theta = 8(1 + \frac{1}{m^2})$. Setting this equal to 18:
$$8\left(1 + \frac{1}{m^2}\right) = 18 \implies 1 + \frac{1}{m^2} = \frac{9}{4} \implies \frac{1}{m^2} = \frac{5}{4} \implies m^2 = \frac{4}{5} \implies m = \frac{2\sqrt{5}}{5}.$$

28. Distance from vertex $(0, 0)$ to focus $(0, 4)$ is $p = 4$. Since the parabola opens upward, the equation is $x^2 = 4py \implies x^2 = 16y$.

8 Closing Thoughts

Coordinate geometry on the AMC 10 and AMC 12 is a straightforward way to turn geometric intuition into simple algebra. Master these core techniques—axis placement, radical axes, line-circle intersections, and standard conic properties—to reliably handle high-numbered AMC coordinate problems and secure your AIME qualification. But please, and I mean it: don't be that bashy kid who uses coordinates when not needed.

9 Resources and References

1. Art of Problem Solving Contest Archive — `https://artofproblemsolving.com/`[cite: 1]

2. MAA AMC 10 / 12 Official Contest Archives — `https://www.maa.org/math-competitions`